Cognitive and emotional factors in mathematics development

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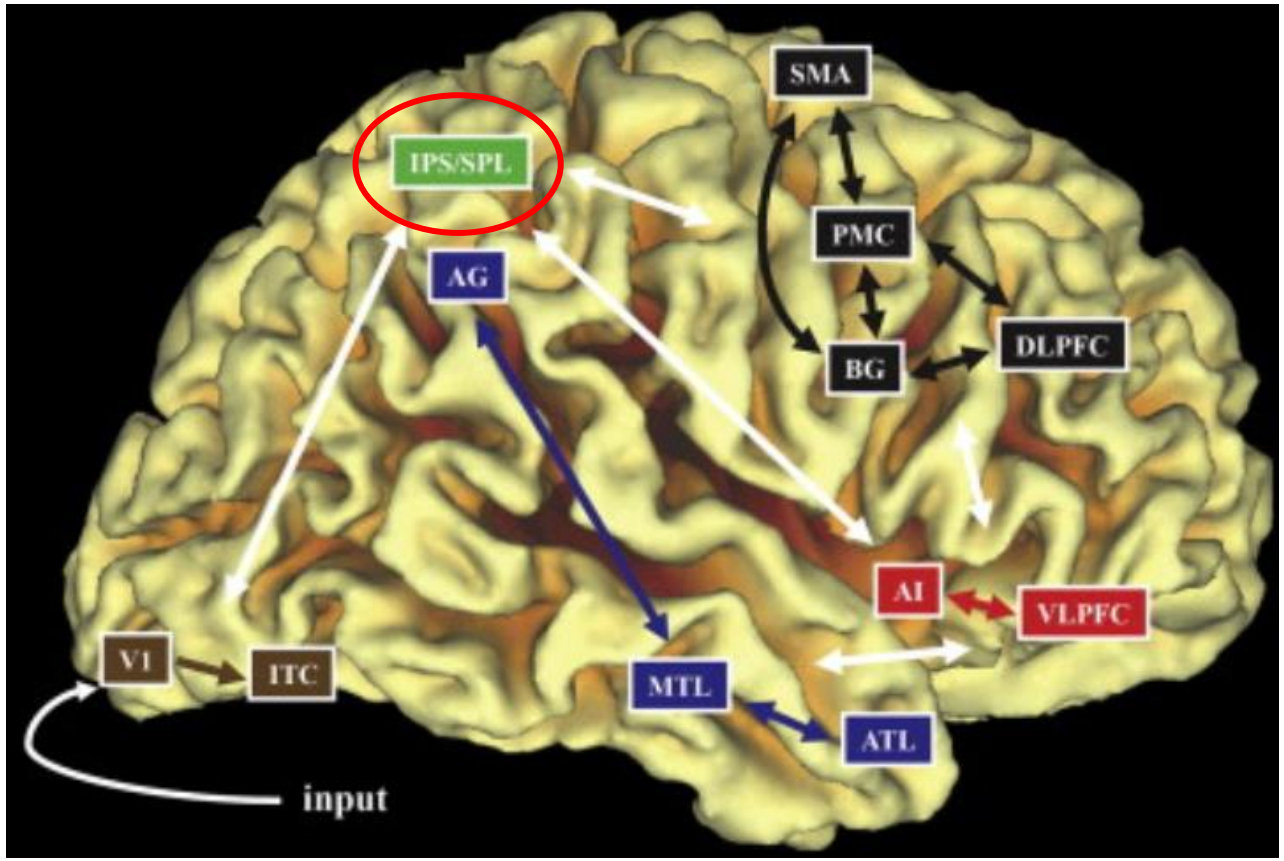
Find out more by reading our **booklet** on Mathematics Anxiety:

Languages: English, **Mandarin** Chinese, Spanish, Italian, **Catalan**, **Hungarian**

A lot of research targets knowing more about the **cognitive skills** necessary to build up mathematics

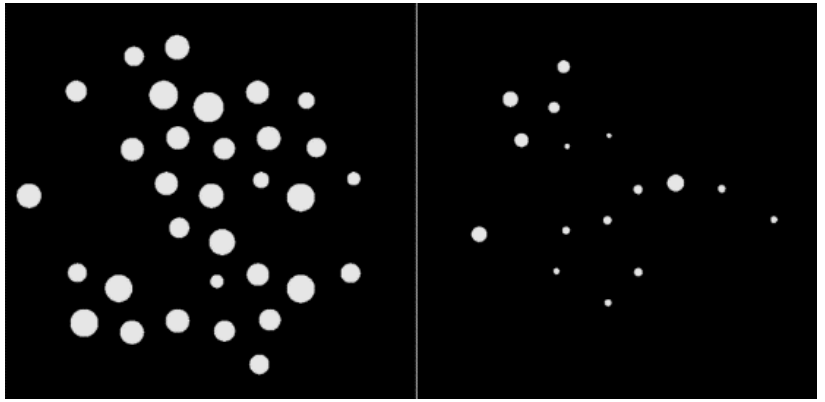
For example, studies about **mathematics development and math learning difficulties**



Menon, Fias, Szűcs, 2016, Trends in Neuroscience and Education

Pinel et al, 2004, Neuron

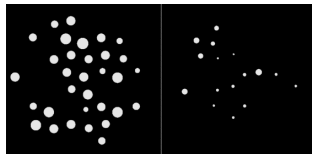
Non-symbolic magnitude comparison task



A large **replication/extension** of our previous study: Standardized math and reading curriculum test predictors in **1254 grade 2, 4, and 6 children** (Ns = 413, 391, 450)

***Null/Alternative**: 0/1 ; **Evidence**: 0=weak ; 1=substantial ; 2=strong ; 3=very strong ; 4=decisive*
Weak: 1-3; substantial: 3-10; strong: 10-30; very strong: 30-100; decisive: >100

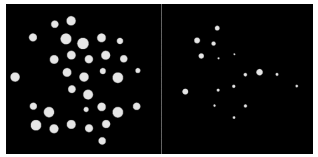
ANS "Approximate number system": **Dot comparison measures**



	Grade 2				Grade 4				Grade 6			
Measure	*r*		zero	partial	*r*		zero	partial	*r*		zero	partial
weber fraction	-0.13|-0.10	*	1+0	0-0	-0.13|-0.10	*	1+0	0-0	-0.18|-0.07	*	1+4	0-1
weber fraction (congruent trials)	-0.13|-0.11	*	1+0	0-0	-0.01|0.01		0-2	0-2	-0.18|-0.10	*	1+3	0-0
weber fraction (incongruent trials)	-0.06|-0.04		0-1	0-2	-0.10|-0.09		0-1	0-1	-0.06|-0.02		0-1	0-2
non-symbolic decision accuracy	0.15|0.09	*	1+1	0-0	0.17|0.13		1+2	1+0	0.16|0.06	*	1+2	0-1
non-symbolic accuracy (congurent trials)	0.12|0.09	*	1+0	0-0	0.08|0.05		0-1	0-1	0.18|0.11	*	1+4	0-0
non-symbolic accuracy (incongruent trials)	0.11|0.05		0-0	0-1	0.20|0.17		1+4	1+2	0.08|-0.02		0-1	0-2
non-symbolic RT	-0.07|-0.06		0-1	0-1	-0.03|-0.00		0-2	0-2	-0.12|-0.09	*	1+0	0-0
non-symbolic RT (congruent trials)	-0.08|-0.06		0-1	0-1	-0.05|-0.01		0-2	0-2	-0.14|-0.12		1+1	1+0
non-symbolic RT (incongruent trials)	-0.06|-0.06		0-1	0-1	-0.02|-0.00		0-2	0-2	-0.10|-0.07		0-0	0-1
symbolic decision accuracy	0.29|0.22		1+4	1+4	0.16|0.14		1+2	1+0	0.16|0.13		1+2	1+0
symbolic distance effect (accuracy)	-0.11|-0.10		0-0	0-0	-0.12|-0.11	*	1+0	0-0	0.01|0.03		0-2	0-2
symbolic RT	-0.21|-0.14		1+4	1+1	-0.17|-0.11	*	1+2	0-0	-0.37|-0.26		1+4	1+4
symbolic distance effect (reaction time)	-0.00|-0.03		0-2	0-2	0.14|0.12		1+0	1+0	0.15|0.11	*	1+1	0-0
spatial short-term memory	0.25|0.17		1+4	1+3	0.13|0.06	*	1+0	0-1	0.23|0.11	*	1+4	0-0
verbal short-term memory	0.29|0.22		1+4	1+4	0.31|0.25		1+4	1+4	0.33|0.31		1+4	1+4
spatial working memory	0.32|0.22		1+4	1+4	0.26|0.17		1+4	1+2	0.41|0.24		1+4	1+4
verbal working memory	0.33|0.25		1+4	1+4	0.27|0.21		1+4	1+4	0.34|0.23		1+4	1+4
Cattel (IQ)	0.38|—	*	1+4	–	0.36|—	*	1+4	–	0.49|—	*	1+4	–
reading composite score (errors)	-0.30|-0.24		1+4	1+4	-0.33|-0.29		1+4	1+4	-0.32|-0.24		1+4	1+4
reading composite score (rate)	0.37|0.34		1+4	1+4	0.38|0.36		1+4	1+4	0.40|0.36		1+4	1+4

w

Correlation of ANS measures and math scores



ANS vs. Math correlations (abs[r]) are **negligible**: $R^2 \leq [r=-0.18]^2 = \sim 0.0324$

(Caviola et al. 2020; **Halberda et al. 2012 (r=-0.13)**; Lyons et al. 2014; Wei et al. 2012; Meta-analysis: Schneider et al. 2017)

Caviola, Colling, Mammarella & Szucs; **2020**

A large **replication/extension** of our previous study: Standardized **math curriculum test** predictors in **1254 grade 2, 4, and 6 children** (Ns = 413, 391, 450)

ANS "Approximate number system": Dot comparison

Symbolic number comp.

STM/WM

CONTROL

Measure	Grade 2 r		zero	partial	Grade 4 r		zero	partial	Grade 6 r		zero	partial
weber fraction	-0.13|-0.10	*	1+0	0-0	-0.13|-0.10	*	1+0	0-0	-0.18|-0.07	*	1+4	0-1
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non-symbolic accuracy (incongruent trials)	0.11|0.05		0-0	0-1	0.20|0.17		1+4	1+2	0.08|-0.02		0-1	0-2
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spatial working memory	0.32|0.22		1+4	1+4	0.26|0.17		1+4	1+2	0.41|0.24		1+4	1+4
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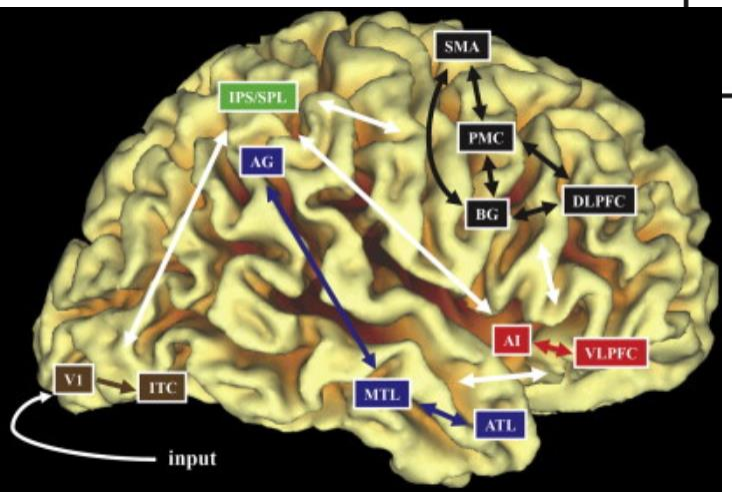
reading composite score (rate)	0.37|0.34		1+4	1+4	0.38|0.36		1+4	1+4	0.40|0.36		1+4	1+4

Symbolic number comparison is skill ***directly relevant*** *to school math.* ***I.e. it is not a measure of a cognitive representation..***

Best **specific** predictors in **regression** models:
Symbolic number comparison and spatial WM
Verbal STM/WM: strongly related to both math and reading.

Caviola, Colling, Mammarella & Szucs; **2020**

Cognitive structure behind numerical competence?



Space

Visual pattern recognition (long-term memory)

Spatial Ability

visual STM | **visual WM**

verbal STM | **verbal WM**

Memory

Inhibition

Selective Attention

Task Switching

Executive Functions

Exec. F. 'Organizing'

Phonological Decoding
Verbal IQ

Fact retrieval (long-term memory)

Language

Spatial Manipulations

Spatial Representations

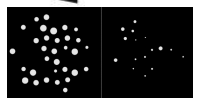
Symbol Manipulation

Symbol Grounding

Symbols

Perhaps ANS?

Magnitude Relations



Verbal Operations

Verbal Representations

Szucs et al. *Developmental Science*, 2014

Szucs *Progress in Brain Research*, 2016

Cognitive difficulties are **not the only** problem related to maths learning…

Lots of students and adults **not only** have **cognitive difficulties** with math **but** are also **afraid** of it..

Math anxiety is fear of learning and doing mathematics.

Mathematics Anxiety ranges from feeling **mild** tension to experiencing **strong** fear of mathematics.

Mathematics Anxiety can be experienced in **school** or in **everyday life**,
e.g. when trying to understand a mortgage.

Mathematics anxiety has negative consequences…

Short-term

Math anxiety leads to **worries** that can occupy students' thoughts during test situations, **negatively affecting math scores**.

Medium-term

Students with high math anxiety may **avoid elective math classes** (even if they are good in math).
Therefore, the math achievement level of these students may **not reach their full potential**.

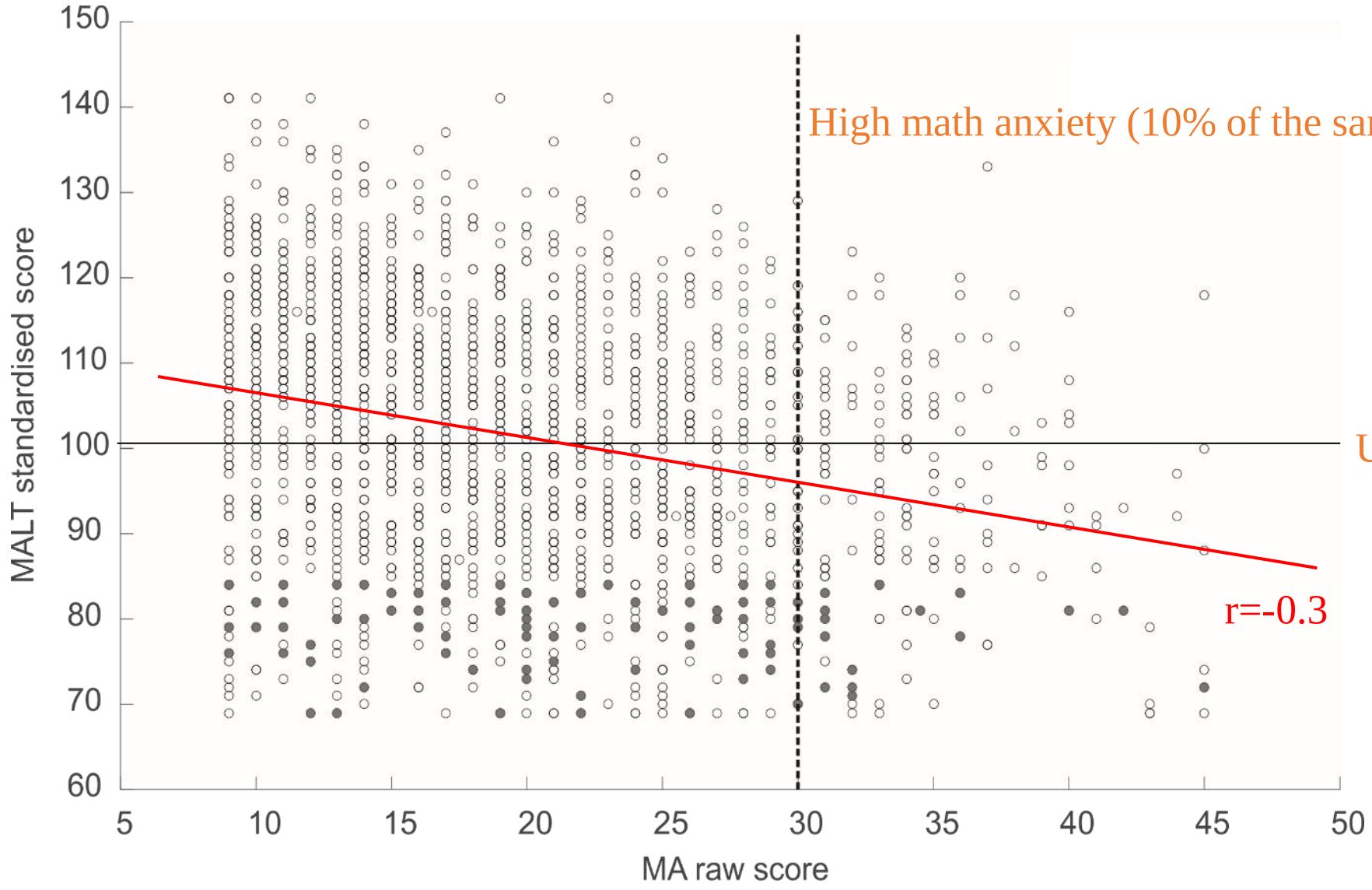
Long-term

Students and adults with high level of math anxiety may **avoid math-related careers** all together.
Females tend to have higher math anxiety than males.
Math anxiety may contribute to the **relative lack of women** in math-heavy careers, e.g. in **STEM** fields.
Math-anxious adults may experience **lesser quality of life**

Children spend **most of their time in school**, they should not be overly anxious about academic life
E.g. **failing to reflect properly** on mortgage payments, gambling risks.

The higher is mathematics anxiety the lower is mathematics achievement in large samples

N=1746, primary and middle school students

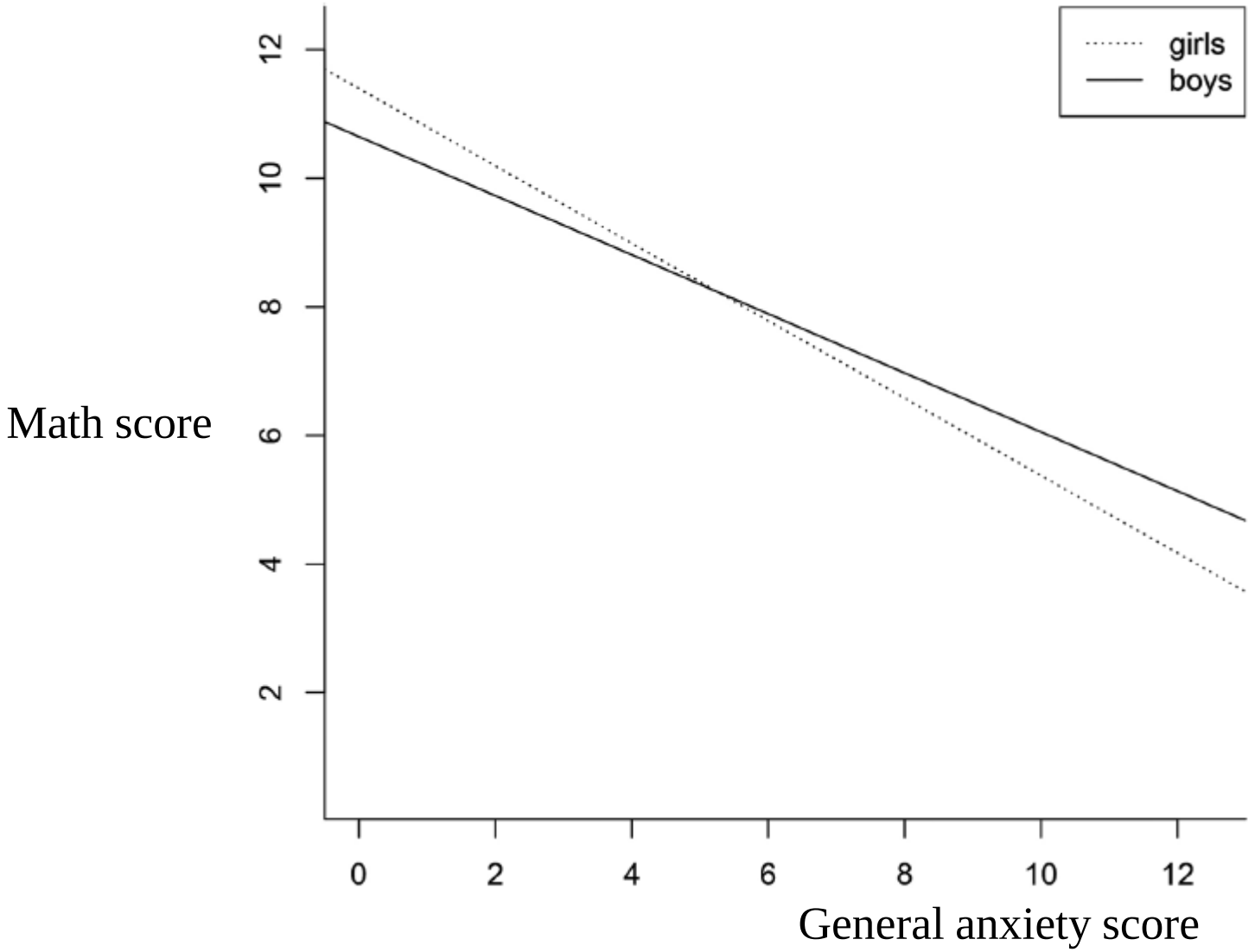


Nuffield Foundation

JAMES S. McDONNELL FOUNDATION

Devine,…Szűcs, 2018; J Educ Psych

General anxiety is correlated with math performance already in Kindergarten
488 **Hungarian kindergarten children**, aged **5.75 to 6.92 years**

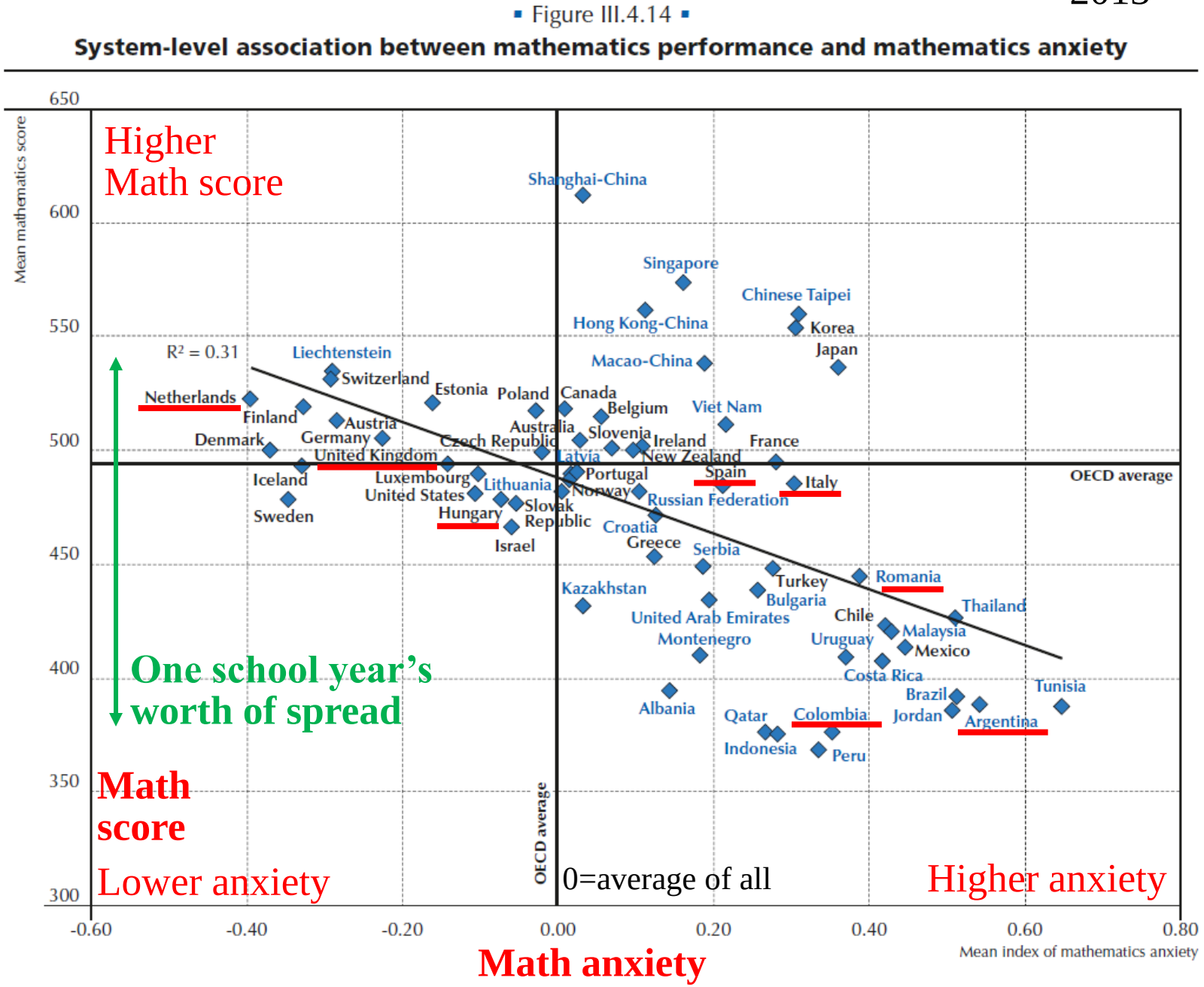


Svraka, Alvarez, Szűcs, 2024

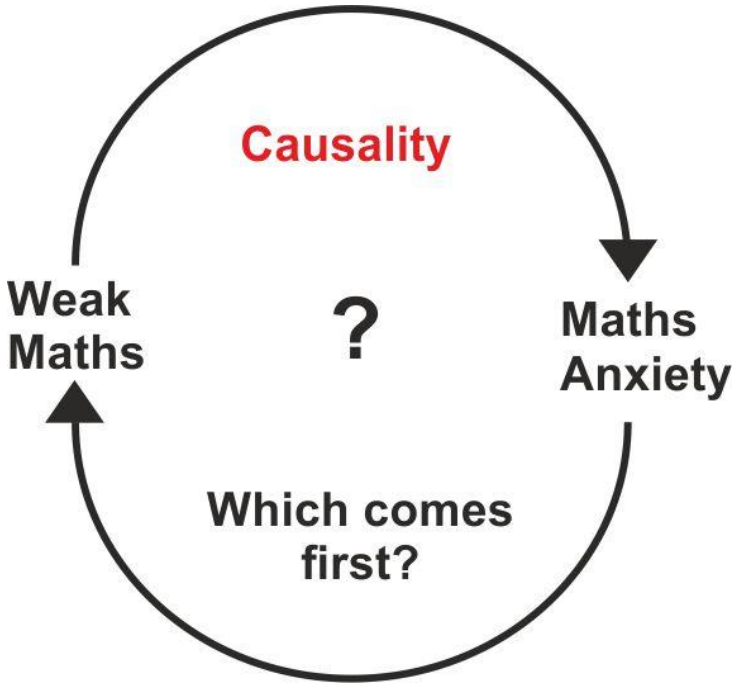
Mathematics anxiety is negatively associated with mathematics scores
PISA, 2012, 15-year-olds

OECD 2013

In **countries** with **high** math anxiety students tend to have **lower** math performance.



How / why is math anxiety generated?



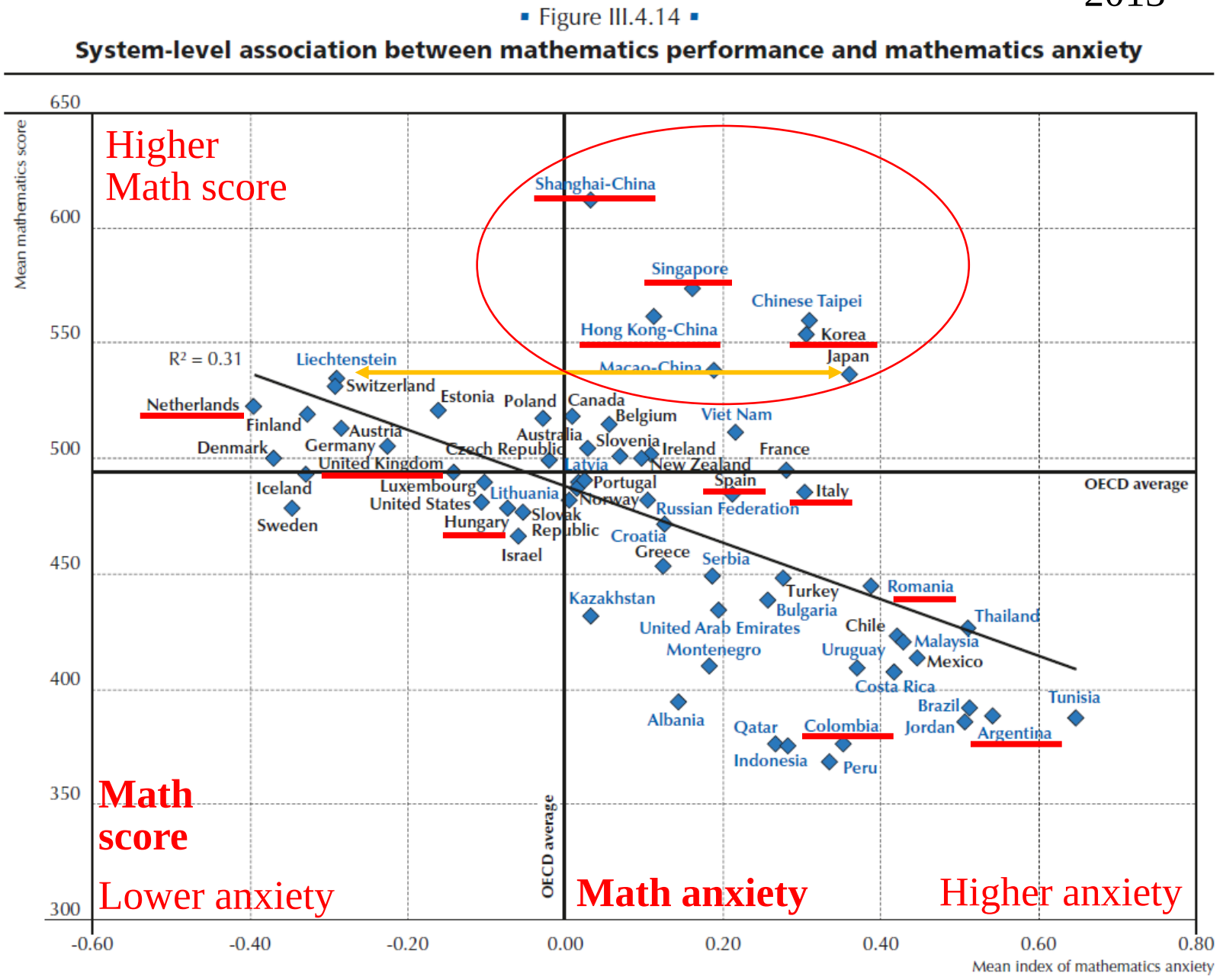
For **review** see: Carey, Hill, Devine & Szűcs, *Frontiers in Psychology*, 2015

Mathematics anxiety is negatively associated with mathematics scores

OECD 2013

In **countries** with **high** math anxiety students tend to have **lower** math performance.

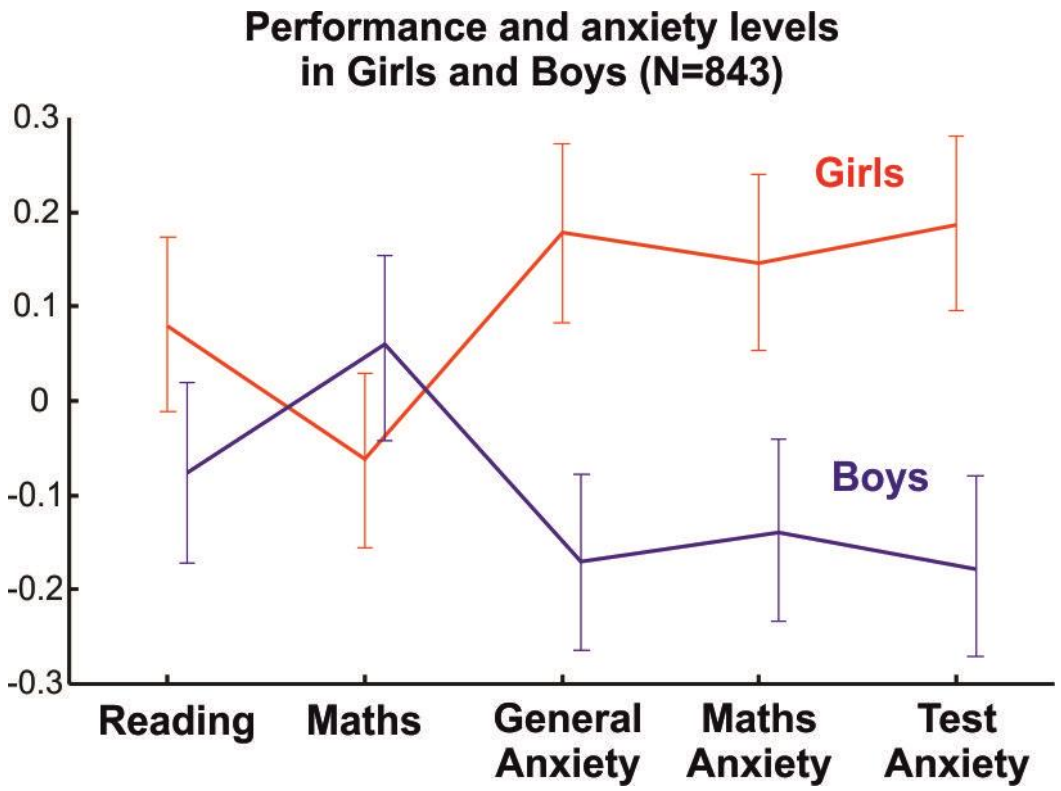
However,
math anxiety is also **(relatively) high** at some observations points with the highest math achievement in the world



In most countries math anxiety is higher in girls than boys <u>even if</u> girls are as good in math as boys

In fact, math anxiety is often much **higher** in girls than in boys in highly **gender equal** countries (gender-equality paradox)

Devine … Szucs et al, 2018; UK; 10-year-olds



Hembree, 1990

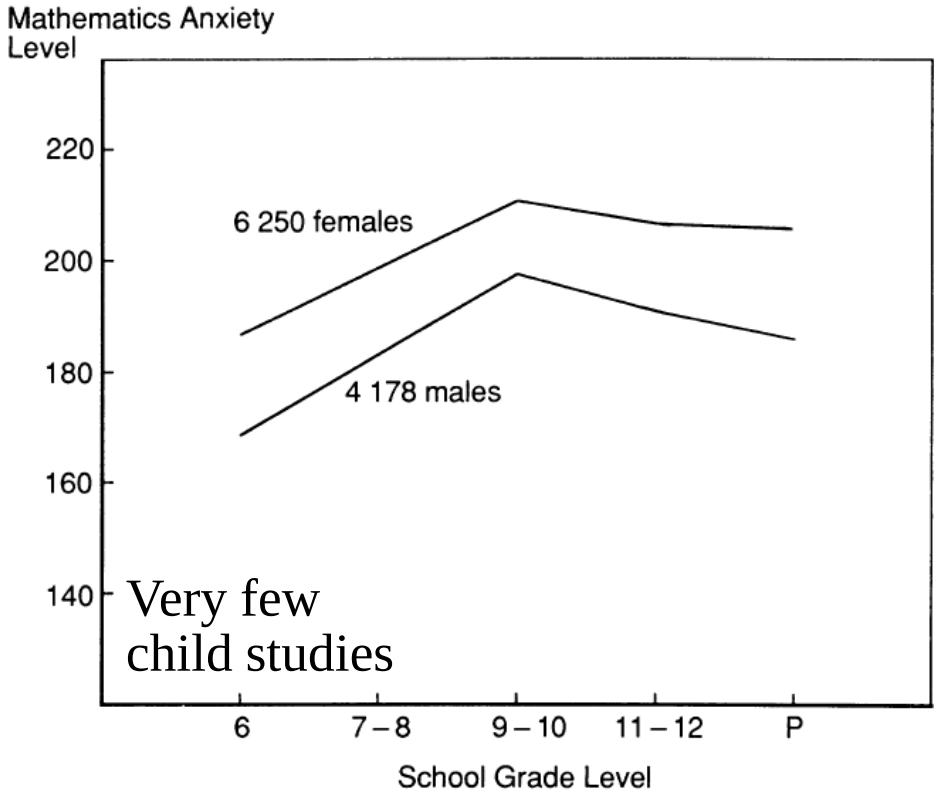
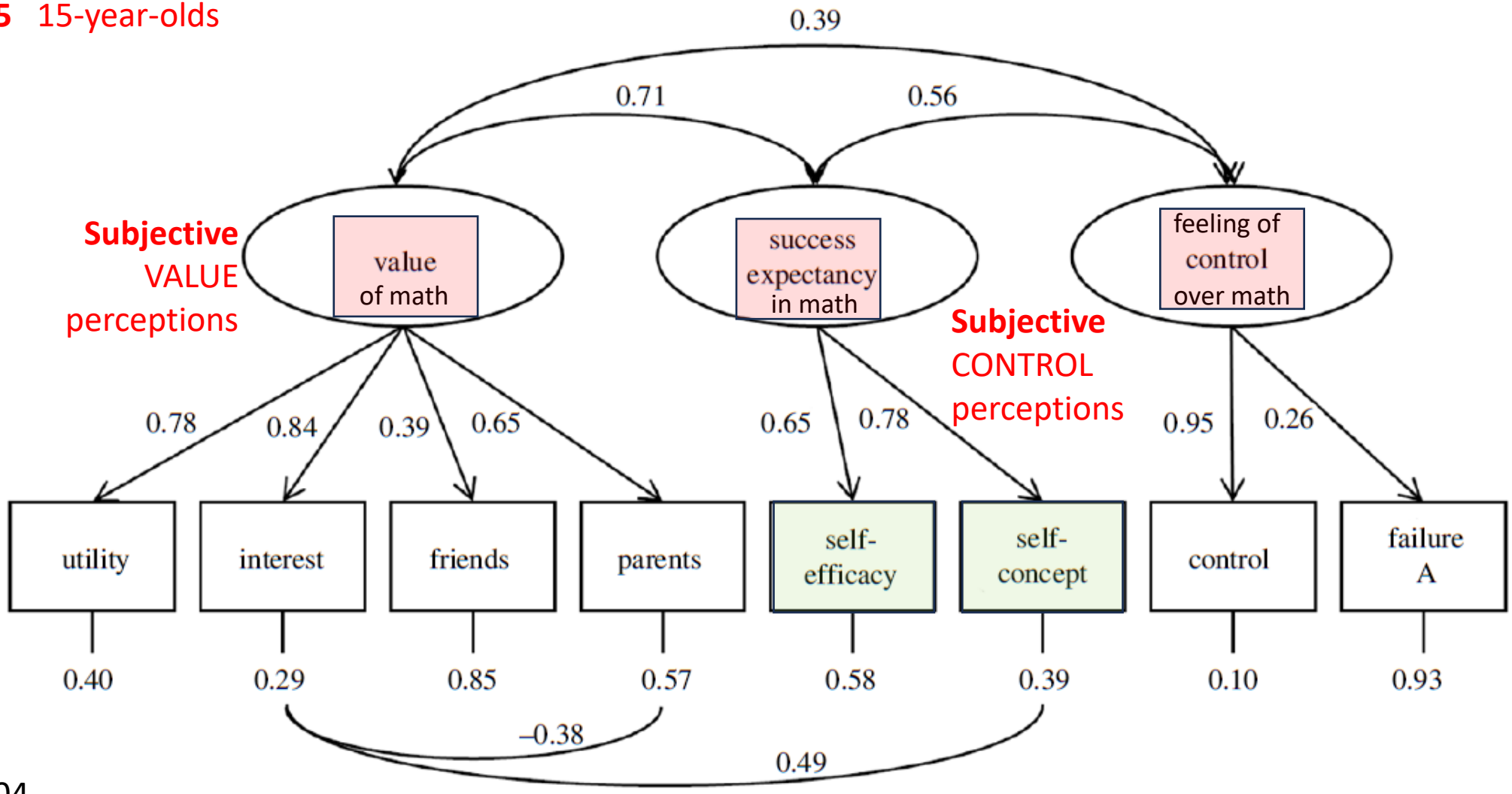


Figure 1. Average mathematics anxiety levels for Grades K–12 and undergraduate.

Maths anxiety and subjective control and value perceptions ← Control-Value and Expectancy-Value theory

Data from PISA 2012
N = **151,745** 15-year-olds



CFA
RMSEA = 0.04
CFI = 0.97
NNFI = 0.96

Szűcs and Toffalini, 2023, Royal Soc Open Sci

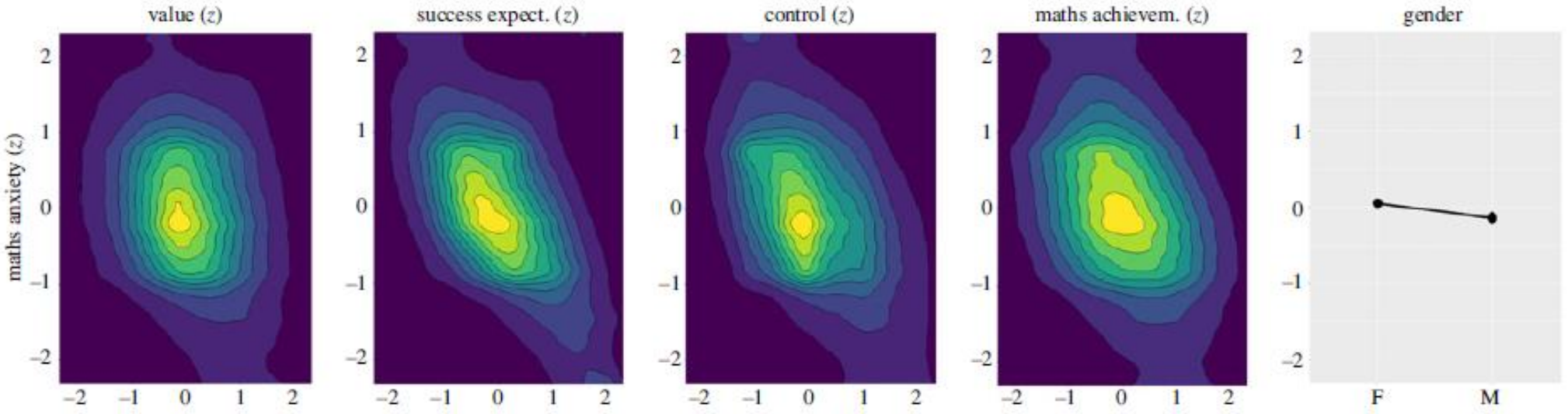


Figure 2. Two-dimensional density plots of MA (vertical axis) and its potential predictors (horizontal axis). The rightmost panel shows mean gender difference in Maths Anxiety. Colour represents density: darkest = (0.00, 0.02], lightest = (0.20, 0.22].

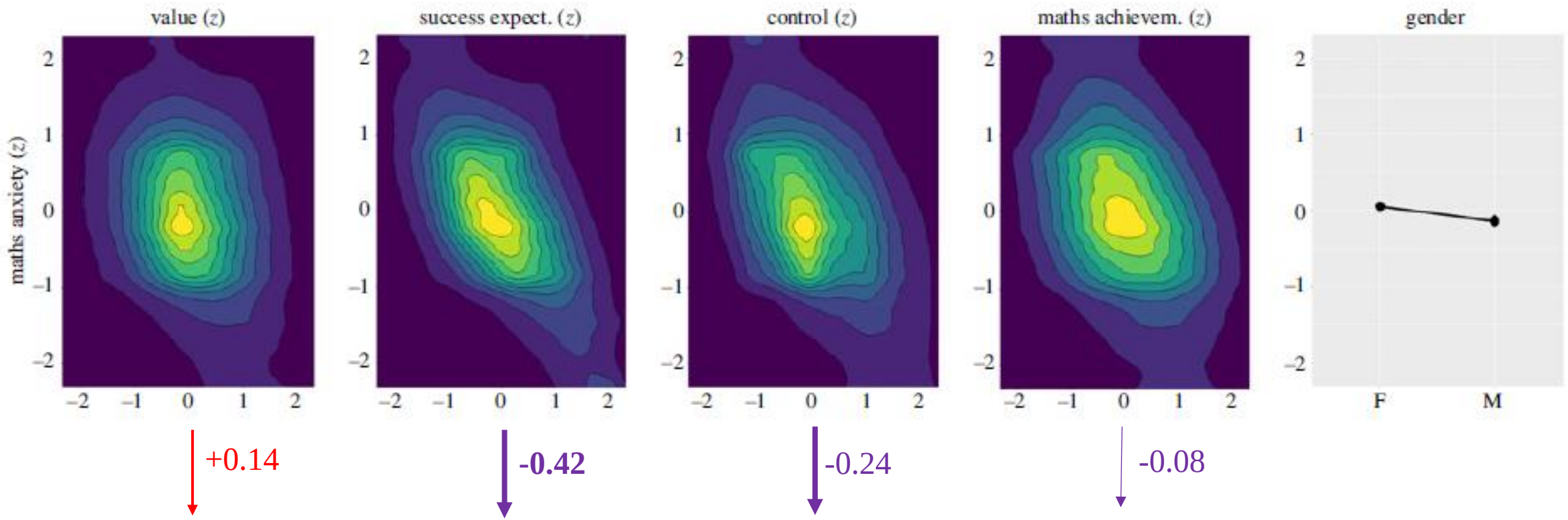
z-scores	1.	2.	3.	4.	5.
		value	Success expectancy	control	
1. maths anxiety	—				
2. value	−0.16	—			
3. success expectancy	−0.47	0.55	—		
4. control	−0.40	0.32	0.43	—	
5. maths achievement	−0.34	0.07	0.44	0.30	—

Subjective self-**perceptions only mildly correlate** with actual achievement

Heterogenity over sampling error; Low: 0-40%

Szűcs and Toffalini, 2023, Royal Soc Open Sci

Linear Mixed Model Coefficients (The impact of a variable when all other ones are fix).

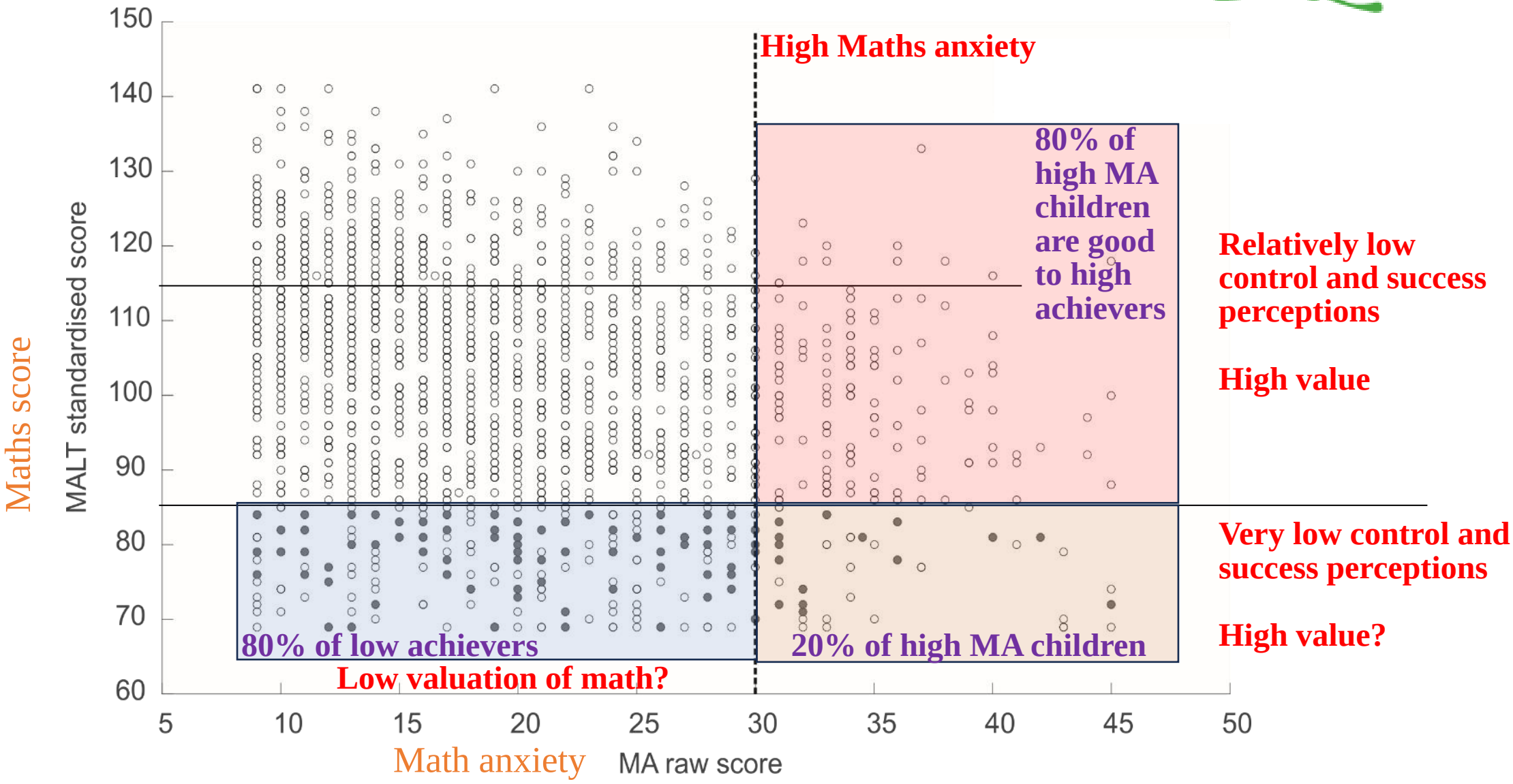


Math Anxiety

Subjective success expectancy, **control** and **value** perceptions
are **more important** than actual math **achievement** in predicting math anxiety

Szűcs and Toffalini, 2023, Royal Soc Open Sci

Emotional and cognitive maths problems dissociate (N=1746)



Devine,…Szucs, **2018**; J Educ Psych

Mathematics anxiety is negatively associated with mathematics scores world-wide

OECD 2013

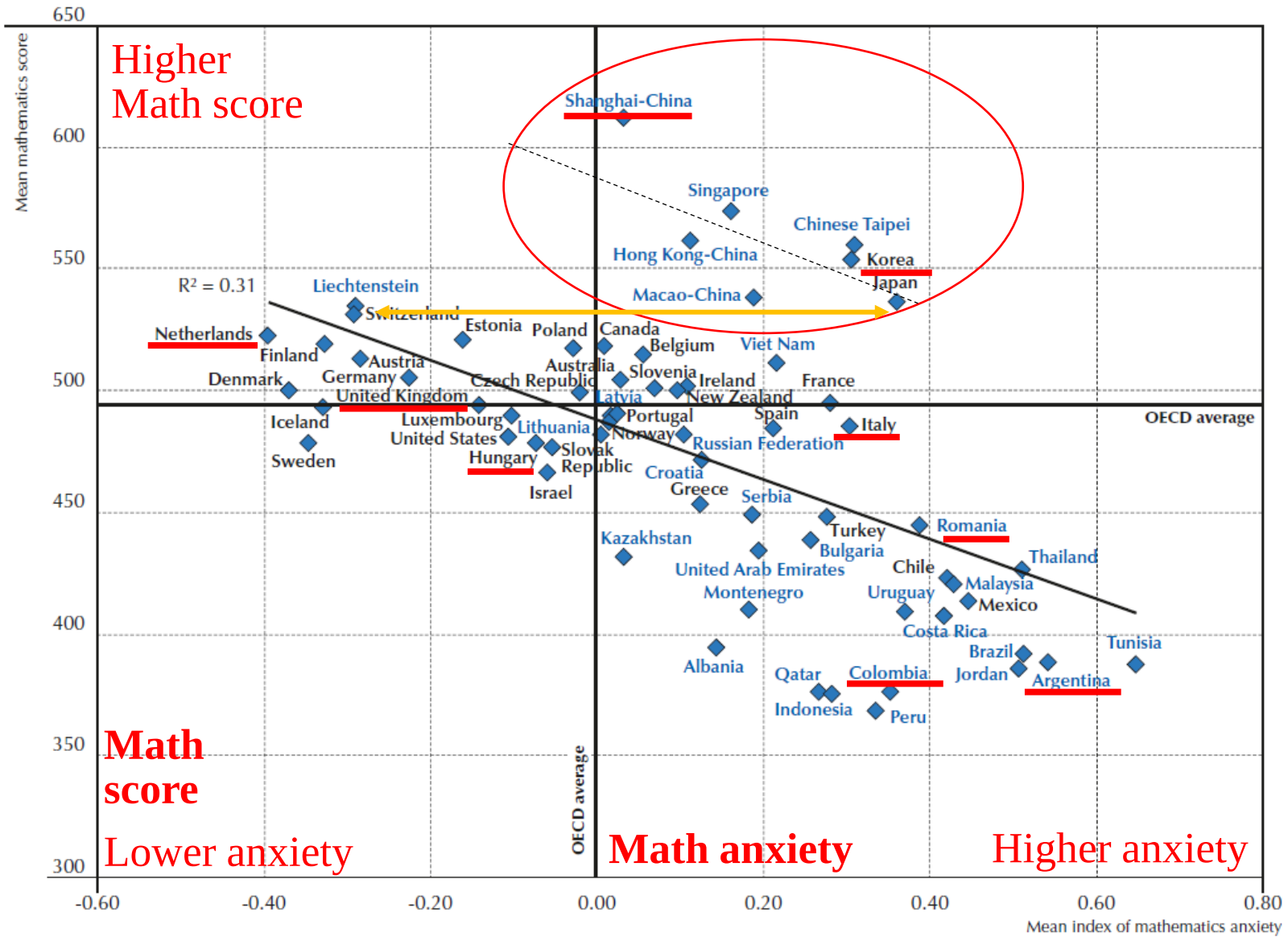
Cultural context must be considered.

Control / Success Expectancy in math
- School, parental, individual **expectations**
- **Competitors**

Value of math
- May be **very high**
  (e.g. due to its 'gate-keeper' function)

▪ Figure III.4.14 ▪

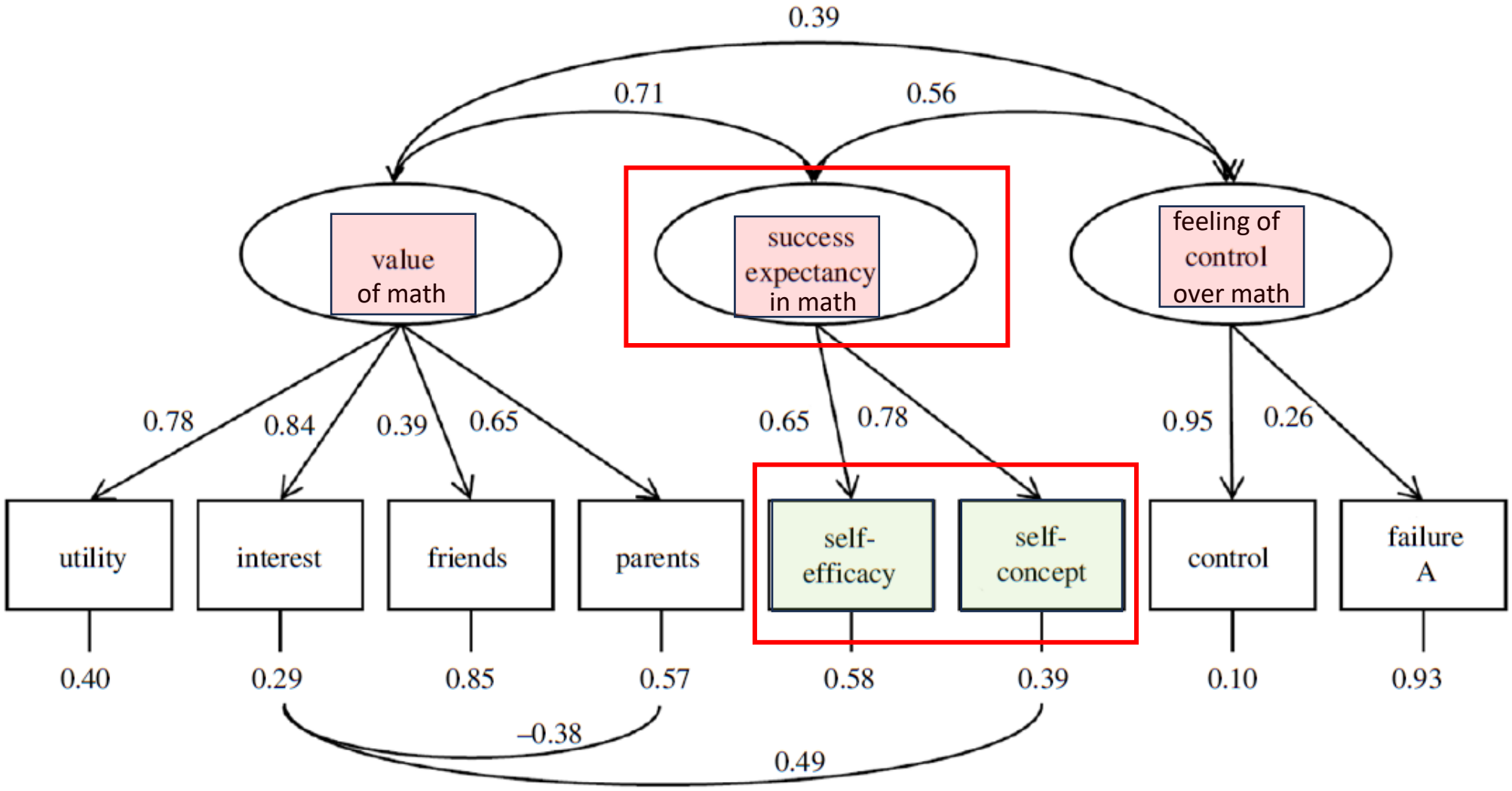
System-level association between mathematics performance and mathematics anxiety



Maths anxiety and subjective control and value perceptions

Data from PISA 2012

N = **151,745** 15-year-olds



CFA

RMSEA = 0.04

CFI = 0.97

NNFI = 0.96

Szűcs and Toffalini, 2023, Royal Soc Open Sci

Research on cognitive skill development often overlooks the importance of subjective factors

Standards of **reference**
To whom do I **compare** myself?

Success/Failure attributions
Success due to **skill or luck**?

Appraisals from **significant others**
Parents, teachers, friends

Mastery experience
Chance to do well

Personal importance of areas
Do I **care** about math?

Physiological/emotional reactions
Do I feel **nervous**?
Do I **like or hate** the teacher?

Bong (2003)

Self-**image**
in domains of life

Self-concept

Self-efficacy

Perceived **efficiency**
in domains of life

Self-esteem

Perceived
self-**valuation**
in domains of life

How to alleviate mathematics anxiety?

Considering SUBJECTIVE feelings of Control, Success Expectancy and Value of math is likely beneficial.

Poorly performing students with high math anxiety

- **Very gradually** increase **exercise** difficulty in order to **build <u>both</u> math skills** AND **confidence** step by step.
- Reward genuine, invested **effort and thinking**, not only achieving correct solution.
- Students should understand that it is **natural** and **inevitable** to make **errors and** they can **learn** from errors.
- Students should understand the **difference** between **doing math** and doing math **quickly** (under time pressure in a test).
- Do **<u>not</u> overemphasize** the VALUE of math.

Well performing students with relatively low confidence in math and with high math anxiety

- Let students **understand** that their anxiety can have **negative** impact on their math performance.
- Let students **understand** that their (relatively good) math performance and their worries about math are **distinct**.
- Put students' (relatively good) performance into **perspective**. They may underestimate their **ability** for doing math.
- Build **confidence** at the **right performance level** (e.g. when increasing expectations towards a student).
- It may be useful to **discuss worries about math** and potential **ways to resolve** these worries.
- Some students may need **coaching** or **counselling** on **anxiety management** (e.g. about talking in front of the class).

Girls with high math anxiety

- Understand the **math gender stereotypes** of students, teachers and parents and their potential **impact** on performance.
- Do **not attribute** success or failure on a math task to gender.
- **Promote** STEM subjects to girls and boys equally at all levels of schooling.
- Let older students to understand their **own** gender stereotypes about math which they may have brought from their family.

Girls often show much higher levels of mathematics anxiety than boys even if they are as good in math as boys

Most studies in most countries found that girls and boys perform **equally** well in math in school.
However, math anxiety is often much **higher** in girls than in boys, **even** in highly gender equal countries.

This **discrepancy in self-reported math anxiety between girls and boys** can be due to **several** reasons:

- There often exist **gender stereotypes** about boys being better in **math and science** than girls.
  Many **parents, teachers and students** share these stereotypes.
  Children can learn these stereotypes very early in life, even before entering school.
- Girls often report **lower self-confidence (lesser feelings of control and relative success expectancy)** in mathematics than boys even if they perform equally well.
  Negative stereotypes about girls and math can **lead** to high math **anxiety** and low **confidence** in math.

- Girls are often more conscientious about school performance: **higher subjective value** of math?

- Girls may be **generally more anxious** than boys.
  For example, often girls **also** have higher **general** anxiety and **test** anxiety than boys.
  Overall higher anxiety levels in girls may **predispose** them to higher levels of math anxiety as well.

- Girls may be **more willing to admit** their anxiety in many cultures than boys.
  Boys are often taught to be **tough** and not admit their anxieties.

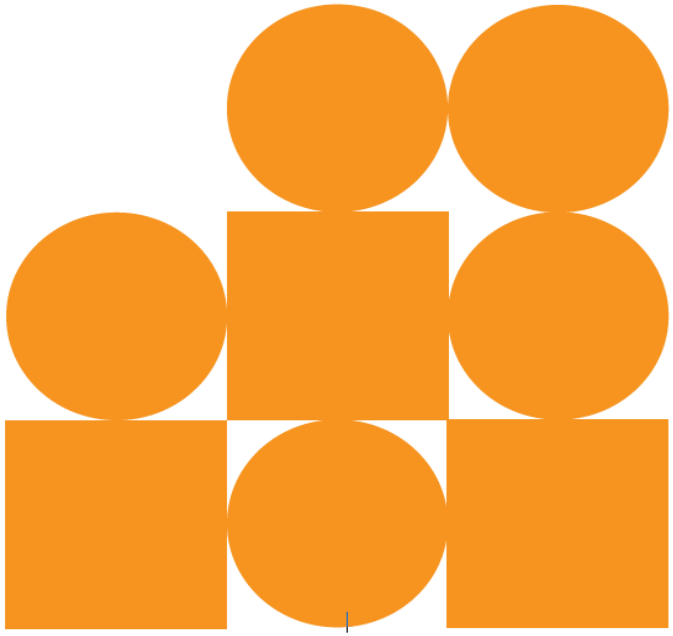
- Girls may be able to **recognize** their anxiety **better** than boys of the same age.
  Girls are often emotionally more **mature** than boys at the same age.

Educational Practices Series 31

Math Anxiety

by Denes Szűcs and Irene C. Mammarella

Szucs & Mammarella, **2020**



Educational Psychology Review
https://doi.org/10.1007/s10648-021-09618-5 **2021**

META-ANALYSIS

Math Performance and Academic Anxiety Forms, from Sociodemographic to Cognitive Aspects: a Meta-analysis on 906,311 Participants

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Cite this article: Szucs D, Toffalini E. 2023 Maths anxiety and subjective perception of control, value and success expectancy in mathematics. *R. Soc. Open Sci.* **10**: 231000. https://doi.org/10.1098/rsos.231000

Maths anxiety and subjective perception of control, value and success expectancy in mathematics **2023**

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ORIGINAL RESEARCH article

Front. Psychol.

Sec. Educational Psychology

Volume 15 - 2024 | doi: 10.3389/fpsyg.2024.1335952

Anxiety predicts math achievement in kindergarten children

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2024